

Package ‘depcoeff’

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Title Dependency Coefficients

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Description Functions to compute coefficients measuring the dependence of two or more than two variables. The functions can be deployed to gain information about functional dependencies of the variables with emphasis on monotone functions. The statistics describe how well one response variable can be approximated by a monotone function of other variables. In regression analysis the variable selection is an important issue. In this framework the functions could be useful tools in modeling the regression function. Detailed explanations on the subject can be found in papers Liebscher (2014) <[doi:10.2478/demo-2014-0004](https://doi.org/10.2478/demo-2014-0004)>; Liebscher (2017) <[doi:10.1515/demo-2017-0012](https://doi.org/10.1515/demo-2017-0012)>; Liebscher (2021): <[https://arfjournals.com/image/catalog/Journals%20Papers/AJSS/No%202%20\(2021\)/4-AJSS_123-150.pdf](https://arfjournals.com/image/catalog/Journals%20Papers/AJSS/No%202%20(2021)/4-AJSS_123-150.pdf)>; Liebscher (2021): Kendall regression coefficient. Computational Statistics and Data Analysis 157. 107140.

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kendr	<i>Kendall regression coefficient</i>
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Description

The function `kendr` evaluates the Kendall regression coefficient. It describes how well the target variable `y` can be fitted by a function of the regressor variables which increases or decreases as the regressors increase.

Usage

```
kendr(x,y,direction=NULL,out=0,outsd=TRUE,eps=0.95)
```

Arguments

<code>x</code>	data matrix of regressor variables
<code>y</code>	data vector of the response variable
<code>direction</code>	vector of length <code>d</code> (<code>d</code> is number of regressors), value 1 refers to regressors leading to increasing <code>y</code> whenever this regressor increases, value -1 refers to regressors leading to decreasing <code>y</code> whenever this regressor increases. If <code>direction=NULL</code> , then all coefficients are computed.
<code>out</code>	value 1: full output, value 0: reduced output of one coefficient that is largest in absolute value
<code>outsd</code>	logical. If <code>TRUE</code> , the estimated standard deviation and confidence intervals are evaluated. If <code>FALSE</code> , the evaluation is suppressed
<code>eps</code>	real value, confidence level.

Value

A list of the coefficients for several directions with components:

dcoeff Kendall regression coefficient

dir direction vector

Pxx fraction of $X \leq \check{X}$ where \check{X} has the same distribution as X

sd standard error

conf confidence interval

References

Eckhard Liebscher (2021). Kendall regression coefficient. Computational Statistics and Data Analysis 157 (2021). 107140

Examples

```
library(MASS)
data <- gilgais
kendr(data[,1:3],data[,4],out=1)
```

kendrm	<i>Multivariate Kendall regression coefficient</i>
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Description

The function kendrm evaluates the multivariate Kendall regression coefficient. It describes how well the response vector y can be fitted by a function of the regressor variables which increases or decreases as the regressors increases.

Usage

```
kendrm(x,y,direction=NULL,out=0,outsd=TRUE,eps=0.95)
```

Arguments

x	data matrix of regressor variables
y	data matrix of the response vector
direction	vector of length d (d is number of regressors), value 1 refers to regressors leading to increasing y whenever this regressor increases, value -1 refers to regressors leading to decreasing y whenever this regressor increases. If direction =NULL, then all coefficients are computed.
out	value 1: full output, value 0: reduced output of one coefficient that is largest in absolute value, value 2: only the largest coefficient
outsd	logical. If TRUE, the estimated standard deviation and confidence intervals are evaluated. If FALSE, the evaluation is suppressed
eps	real value, confidence level.

Value

A list of the coefficients for several directions with components:

dcoeff Kendall regression coefficient

dir direction vector

Pxx fraction of $X \leq \check{X}$ where \check{X} has the same distribution as X

sd standard deviation

conf confidence interval

References

Eckhard Liebscher (2026). Kendall regression coefficient revisited.

Examples

```
library(faraway)
y<- fat[,c("brozek","siri")]
x<- fat[,c("weight","adipos","abdom","hip")]
kendrm(x,y,direction=NULL,out=1,outsd=FALSE)
kendrm(x,y,direction=NULL,out=0,outsd=TRUE)
```

kendrs

Kendall regression coefficient for split domains

Description

The function `kendrs` evaluates the multivariate Kendall regression coefficient for two regressors and split regressor region. It describes how well the response variable can be fitted in each split region by a function which increases or decreases as the regressors increase.

Usage

```
kendrs(x,y,splitp=NULL)
```

Arguments

<code>x</code>	data matrix of regressor variables with two columns,
<code>y</code>	data vector of the response variable
<code>splitp</code>	vector of length 2 of the splitting points, If <code>p1</code> is the first component of this vector, then the point splits the domain of the first regressor into a left region of fraction <code>p1</code> of data items and a right region of the remaining data items. The same is done for the second regressor. As the result we obtain 4 subregions of the regressor domain. default= <code>c(0.5,0.5)</code>

Value

A list of Kendall regression coefficients for the 4 split regions (11=left lower region, 12=left upper region...) with components

dcoeff++ split coefficient ++

dcoeff+- split coefficient +-

totalcoeff total coefficient

directions optimal directions

direction ++ means that y increases whenever both regressors increases direction +- means that y increases whenever the first regressor increases and the other regressor decreases..etc.

References

Eckhard Liebscher (2021). Kendall regression coefficient. Computational Statistics and Data Analysis 157 (2021). 107140

Examples

```
library(MASS)
data<- gilgais
kendrs(data[,1:2],data[,3],splitp=c(0.4,0.6))
```

kendtaum

Multivariate Kendall's tau

Description

The function kendtaum evaluates the multivariate Kendall's tau coefficient. It describes the dependence of the variables in the data matrix

Usage

```
kendtaum(x, outsd=TRUE, eps=0.95)
```

Arguments

x	data matrix of regressor variables
outsd	logical. If TRUE, the estimated standard deviation and confidence intervals are evaluated. If FALSE, the evaluation is suppressed
eps	real value, confidence level.

Value

A list of the coefficients for several directions with components:

dcoeff multivariate Kendall's tau

sd standard error

conf confidence interval

References

Schmid, F.; Schmidt, R.; Blumentritt, T.; Gaißer, S.; Ruppert, M. (2010). Copula-based measures of multivariate association. in F. Durante, W. Härdle, P. Jaworski, T. Rychlik (eds.) Copula Theory and Its Applications. Springer Berlin, 2010.

Examples

```
library(MASS)
data <- gilgais
kendtaum(data[,1:4], outsd=TRUE)
```

spearr	<i>Spearman regression coefficient</i>
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Description

The function `spearr` evaluates the multivariate Spearman regression coefficient. It describes how well the target variable `y` can be fit by a function of regressor variables which is increasing w.r.t. some regressors and decreasing w.r.t. the other regressors.

Usage

```
spearr(x,y,direction=NULL,out=0)
```

Arguments

<code>x</code>	data matrix of regressor variables
<code>y</code>	data vector of the target variable
<code>direction</code>	vector of length <code>d</code> (<code>d</code> is number of regressors), value 1 refers to regressors leading to increasing <code>y</code> whenever this regressor increases, value -1 refers to regressors leading to decreasing <code>y</code> whenever this regressor increases. If <code>direction=NULL</code> , then all coefficients are computed.
<code>out</code>	value 1: full output, value 0: reduced output, only coefficient that is largest in absolute value

Value

A list containing

dcoeff Spearman regression coefficient

dir direction vector

References

Eckhard Liebscher (2021). On a multivariate version of Spearman's correlation coefficient for regression: Properties and Applications. Asian Journal of Statistical Sciences 1, No. 2, 123-150.

Examples

```
library(MASS)
data <- gilgais
spearr(data[,1:3],data[,4],out=1)
```

spearrs

*Spearman regression coefficient for split domains***Description**

The function `spearrs` evaluates the multivariate Spearman regression coefficient for two regressors and split regressor region. It describes how well the response variable can be fitted in each split region by a function which increases or decreases as the regressors increase.

Usage

```
spearrs(x,y,splitp=NULL)
```

Arguments

<code>x</code>	data matrix of regressor variables with two columns,
<code>y</code>	data vector of the target variable
<code>splitp</code>	vector of length 2 of the splitting points, If <code>p1</code> is the first component of this vector, then the point splits the domain of the first regressor into a left region of fraction <code>p1</code> of data items and a right region of the remaining data items. The same is done for the second regressor. As the result we obtain 4 subregions of the regressor domain. default= <code>c(0.5,0.5)</code>

Value

A list of Spearman regression coefficients for the 4 split regions (11=left lower region, 12=left upper region...) with components:

dcoeff++ split coefficient ++

dcoeff+- split coefficient +-

totalcoeff total coefficient

directions optimal directions for the several split regions

direction ++ means that `y` increases whenever both regressors increases direction +- means that `y` increases whenever the first regressor increases and the other regressor decreases..etc.

References

Eckhard Liebscher (2021). On a multivariate version of Spearman's correlation coefficient for regression: Properties and Applications. Asian Journal of Statistical Sciences 1, No. 2, 123-150.

Examples

```
library(MASS)
data<- gilgais
spearrs(data[,1:2],data[,3],splitp=c(0.4,0.6))
```

xic *Xi dependence coefficient*

Description

xic is a function to evaluate the xi dependence coefficient (one interval) of two random variables x and y which is based on the copula. Two specific coefficients are available: the power coefficient and the Huber function coefficient.

Usage

```
xic(x,y,method="power",methodF=1,parH=0.5,parp=1.5,outsd=TRUE,eps=0.95)
```

Arguments

x, y	data vectors of the two variables whose dependence is analysed.
method	list of names of the coefficients: "power" stands for the power function coefficient, "Huber" means the Huber function coefficient. If "all" is assigned to method then all methods are used.
methodF	value 1,2 or 3 refers to several methods for computation of the distribution function values, 1 is the default value.
parH	parameter of the Huber function (default 0.5). Valid values for parH are between 0 and 1.
parp	parameter of the power function (default 1.5). The parameter has to be positive.
outsd	logical. If TRUE, the estimated standard deviation and confidence intervals are evaluated. If FALSE, the evaluation is suppressed
eps	real value, confidence level.

Details

Let X_1, \dots, X_n be the sample of the X variable. Formulas for the estimators of values $F(X_i)$ of the distribution function: methodF = 1 $\rightarrow \hat{F}(X_i) = \frac{1}{n}\text{rank}(X_i)$ methodF = 2 $\rightarrow \hat{F}^1(X_i) = \frac{1}{n+1}\text{rank}(X_i)$ methodF = 3 $\rightarrow \hat{F}^2(X_i) = \frac{1}{\sqrt{n^2-1}}\text{rank}(X_i)$ The values of the distribution function of Y are treated analogously.

Value

A list of the following vectors:

power zeta coefficient with power function, standard error, confidence interval

Huber zeta coefficient with Huber function, standard error, confidence interval

The zeta dependence coefficient of two random variables is bounded by 1. The higher the value the stronger is the dependence.

References

Eckhard Liebscher (2014). Copula-based dependence measures. *Dependence Modeling 2* (2014), 49-64

Examples

```
library(MASS)
data<- gilgais
xic(data[,1],data[,2])
```

zetaC	<i>Zeta dependence coefficient</i>
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Description

zetaC is a function to evaluate the zeta dependence coefficient (one interval) of two random variables x and y which is based on the copula. Four specific coefficients are available: the Spearman coefficient, Spearman's footrule, the power coefficient and the Huber function coefficient.

Usage

```
zetaC(x,y,method="Spearman",methodF=1,parH=0.5,parp=1.5,outsd=TRUE,eps=0.95)
```

Arguments

x, y	data vectors of the two variables whose dependence is analysed.
method	list of names of the coefficients: "Spearman" stands for the Spearman coefficient, "footrule" means Spearman's footrule, "power" stands for the power function coefficient, "Huber" means the Huber function coefficient. If "all" is assigned to method then all methods are used.
methodF	value 1,2 or 3 refers to several methods for computation of the distribution function values, 1 is the default value.
parH	parameter of the Huber function (default 0.5). Valid values for parH are between 0 and 1.
parp	parameter of the power function (default 1.5). The parameter has to be positive.
outsd	logical. If TRUE, the estimated standard deviation and confidence intervals are evaluated. If FALSE, the evaluation is suppressed
eps	real value, confidence level.

Details

Let X_1, \dots, X_n be the sample of the X variable. Formulas for the estimators of values $F(X_i)$ of the distribution function: $\text{methodF} = 1 \rightarrow \hat{F}(X_i) = \frac{1}{n} \text{rank}(X_i)$ $\text{methodF} = 2 \rightarrow \hat{F}^1(X_i) = \frac{1}{n+1} \text{rank}(X_i)$ $\text{methodF} = 3 \rightarrow \hat{F}^2(X_i) = \frac{1}{\sqrt{n^2-1}} \text{rank}(X_i)$ The values of the distribution function of Y are treated analogously.

Value

A list of the following vectors:

Spearman zeta Spearman coefficient, standard error, confidence interval

footrule zeta footrule coefficient, standard error, confidence interval

power zeta coefficient with power function, standard error, confidence interval

Huber zeta coefficient with Huber function, standard error, confidence interval

The zeta dependence coefficient of two random variables is bounded by 1. The higher the value the stronger is the dependence.

References

Eckhard Liebscher (2014). Copula-based dependence measures. *Dependence Modeling 2* (2014), 49-64

Examples

```
library(MASS)
data<- gilgais
zetac(data[,1],data[,2],method="all")
```

zetaci

Zeta coefficient of piecewise monotonicity with split domain

Description

The function `zetaci` evaluates the coefficient of piecewise monotonicity of variables x and y where the x -domain is split into a fixed number of intervals.

Usage

```
zetaci(x,y,a,method="Spearman",methodF=1,parH=0.5,parp=1.5)
```

Arguments

x, y	data vectors of the two variables whose dependence is analysed.
a	vector of fractions $a_i, 0 < a_i < a_{i+1} < 1$ for the splitting. A fraction of $a_1, a_2 - a_1, a_3 - a_2 \dots$ of data points are in the corresponding split region. The number of split regions is equal to the length of a plus 1.
method	value (default "Spearman")
methodF	value 1,2 or 3 refers to several methods for computation of the distribution function values, 1 is the default value.
parH	parameter of the Huber function (default 0.5). Valid values for parH are between 0 and 1.
parp	parameter of the power function (default 1.5). The parameter has to be positive.

Details

Let X_1, \dots, X_n be the sample of the X variable. Formulas for the estimators of values $F(X_i)$ of the distribution function: methodF = 1 $\rightarrow \hat{F}(X_i) = \frac{1}{n} \text{rank}(X_i)$ methodF = 2 $\rightarrow \hat{F}^1(X_i) = \frac{1}{n+1} \text{rank}(X_i)$ methodF = 3 $\rightarrow \hat{F}^2(X_i) = \frac{1}{\sqrt{n^2-1}} \text{rank}(X_i)$ The values of the distribution function of Y are treated analogously.

Value

A list of zeta dependence coefficients of piecewise monotonicity of two random variables containing the following elements:

Spearman zeta Spearman plusminus/minusplus coefficient

footrule zeta Spearman's footrule plusminus/minusplus coefficient

power zeta power plusminus/minusplus coefficient

Huber zeta Huber plusminus/minusplus coefficient

References

Eckhard Liebscher (2017). Copula-based dependence measures for piecewise monotonicity. *Dependence Modeling* 5 (2017), 198-220

Examples

```
library(MASS)
x<- seq(0,1.0,by=0.01)
eps<- rnorm(length(x),sd=0.05)
y<- x-2*(x>=0.4)*(x-0.4)+4*(x>=0.75)*(x-0.75)+eps
zetaci(x,y, a=c(0.25, 0.5, 0.75))
```

zetapm

*Zeta dependence coefficient of piecewise monotonicity***Description**

zetapm is a function to evaluate the zeta dependence coefficients of piecewise monotonicity of two random variables x and y which is based on the copula. The regressor domain (domain of x) is split into two parts. The function searches for the optimal splitting point to obtain maximum dependence. The main part of the function is coded as C++ procedure

Usage

```
zetapm(x,y,amin=0.25,method="all",methodF=1,parp=1.5,parH=0.5)
```

Arguments

<code>x, y</code>	data vectors of the two variables whose dependence is analysed.
<code>amin</code>	minimum fraction of sample items to be used for one split region
<code>method</code>	vector of chosen special coefficients: Spearman...Spearman coefficient footrule...Spearman's footrule power...power coefficient Huber...Huber function coefficient, "all" refers to all coefficients
<code>methodF</code>	value 1,2 or 3 refers to several methods for computation of the distribution function values, 1 is the default value.
<code>parp</code>	parameter of the power function (default 1.5). The parameter has to be positive.
<code>parH</code>	parameter of the Huber function (default 0.5). Valid values for parH are between 0 and 1.

Details

Let X_1, \dots, X_n be the sample of the X variable. Formulas for the estimators of values $F(X_i)$ of the distribution function: $\text{methodF} = 1 \rightarrow \hat{F}(X_i) = \frac{1}{n}\text{rank}(X_i)$ $\text{methodF} = 2 \rightarrow \hat{F}^1(X_i) = \frac{1}{n+1}\text{rank}(X_i)$ $\text{methodF} = 3 \rightarrow \hat{F}^2(X_i) = \frac{1}{\sqrt{n^2-1}}\text{rank}(X_i)$ The values of the distribution function of Y are treated analogously.

Value

A list of zeta dependence coefficients of piecewise monotonicity of two random variables containing the following elements:

- plusminuscoeff** zeta plusminus coefficients: Spearman, footrule, power, Huber coefficients if chosen
- position1** positions of the optimal split points within the x-value (zeta plusminus coefficients)
- minuspluscoeff** zeta minuspluscoeff coefficients: Spearman, footrule, power, Huber coefficients if chosen
- position2** positions of the optimal split points within the x-value (zeta minusplus coefficients)

References

Eckhard Liebscher (2017). Copula-based dependence measures for piecewise monotonicity. *Dependence Modeling* 5 (2017), 198-220

Examples

```
x<- seq(0,1.0,by=0.01)
eps<- rnorm(length(x),sd=0.05)
y<- x-2*(x>=0.4)*(x-0.4)+eps
zetapm(x,y,amin=0.2,method="all",methodF=1,parp=1.5,parH=0.5)
```

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